1 Syntax

$$
\begin{aligned}
e ::= {} & c \in \mathbb{Z} \\
| {} & \mathsf{x} \in \mathit{Var} \\
| {} & e_1 + e_2 \\
| {} & \lambda \mathsf{x}.s \\
| {} & e_1\,(e_2) \\
| {} & \mathit{alloc} \\
| {} & e.\mathtt{f} \\
| {} & \mathtt{f}\ \mathit{in}\ e
\end{aligned}
\qquad
\begin{aligned}
e_e ::= {} & \cdot +_1 e \\
| {} & \cdot +_2 \cdot \\
| {} & @_1\,(e_2) \\
| {} & @_2 \\
| {} & @_3 \\
| {} & .\mathtt{f} \\
| {} & \mathtt{f}\ \mathit{in}_1 \cdot
\end{aligned}
\qquad
\begin{aligned}
s \in \mathit{stat} ::= {} & \mathit{skip} \\
| {} & s_1; s_2 \\
| {} & \mathsf{x} := e \\
| {} & \mathit{if}\ (e > 0)\ s_1\, s_2 \\
| {} & \mathit{while}\ (e > 0)\ s \\
| {} & \mathit{return}\, e \\
| {} & e_1.\mathtt{f} := e_2 \\
| {} & \mathit{delete}\ e.\mathtt{f}
\end{aligned}
\qquad
\begin{aligned}
s_e ::= {} & \mathsf{x} :=_1 \cdot \\
| {} & \cdot ;_1 s_2 \\
| {} & \mathit{if}_1\, s_1\, s_2 \\
| {} & \mathit{while}_1\ (e > 0)\ s \\
| {} & \mathit{while}_2\ (e > 0)\ s \\
| {} & \mathit{return}_1 \cdot \\
| {} & .\mathtt{f} :=_1 e_2 \\
| {} & .\mathtt{f} :=_2 \cdot \\
| {} & \mathit{delete}_1\ .\mathtt{f}
\end{aligned}
$$

2 Semantics

2.1 Expressions

RED-CONST(c)
$$\frac{}{H_e, \ell_e, \ell_c, c \Downarrow H_e, \ell_e, c}$$

RED-VAR-LOCAL(x)
$$\frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, \ell_c\,[\mathsf{x}]} \quad \mathsf{x} \in \mathit{dom}\,(H_e\,[\ell_c])$$

RED-VAR-GLOBAL(x)
$$\frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, E\,[\mathsf{x}]} \quad \mathsf{x} \in \mathit{dom}\,(H_e\,[\ell_e]) \wedge \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_c])$$

RED-VAR-UNDEF(x)
$$\frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow \mathit{err}} \quad \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_e]) \wedge \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_c])$$

RED-ADD(e_1, e_2)
$$\frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, \cdot +_1 e_2 \Downarrow r'}{H_e, \ell_e, \ell_c, e_1 + e_2 \Downarrow r'}$$

RED-ADD-1(e_2)
$$\frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad v_1, r, \cdot +_2 \cdot \Downarrow r'}{\ell_c, (H_e, \ell_e, v_1), \cdot +_1 e_2 \Downarrow r'}$$

RED-ADD-2
$$\frac{}{v_1, (H_e, \ell_e, v_2), \cdot +_2 \cdot \Downarrow H_e, \ell_e, v_1 + v_2}$$

$$\textsc{red-lambda}(\mathsf{x}, s)\quad \frac{}{H_e, \ell_e, \ell_c, \lambda \mathsf{x}.s \Downarrow H_e, \ell_e, (\ell_c, \lambda \mathsf{x}.s)}$$

$$\textsc{red-app}(e_1, e_2)\quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, @_1(e_2) \Downarrow r'}{H_e, \ell_e, \ell_c, e_1(e_2) \Downarrow r'}$$

$$\textsc{red-app-1}(e_2)\quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad \ell_c', \mathsf{x}, s, r, @_2 \Downarrow r'}{\ell_c, (H_e, \ell_e, (\ell_c', \lambda \mathsf{x}.s)), @_1(e_2) \Downarrow r'}$$

$$\textsc{red-app-2}(s)\quad \frac{\begin{array}{c}\ell_c' = \mathit{fresh}(H_e) \qquad C = H_e[\ell_c] \\ H_e[\ell_c' \leftarrow C[\mathsf{x} \leftarrow v]], \ell_e, \ell_c', s \Downarrow r \qquad r, @_3 \Downarrow r'\end{array}}{\ell_c, \mathsf{x}, s, (H_e, \ell_e, v), @_2 \Downarrow r'}$$

$$\textsc{red-app-3-ret}\quad \frac{}{\mathit{ret}(H_e, \ell_e, v), @_3 \Downarrow H_e, \ell_e, v}$$

$$\textsc{red-app-3-no-ret}\quad \frac{}{H_e, \ell_e, \ell_c, @_3 \Downarrow \mathit{err}}$$

$$\textsc{red-new-obj}\quad \frac{\ell = \mathit{fresh}(H)}{H, H_e, \ell_e, \ell_c, \mathit{alloc} \Downarrow H[\ell \leftarrow \{\}], H_e, \ell_e, \ell}$$

$$\textsc{red-field}(e, \mathsf{f})\quad \frac{H, H_e, \ell_e, \ell_c, e \Downarrow r \qquad r, .\mathsf{f} \Downarrow r'}{H, H_e, \ell_e, \ell_c, e.\mathsf{f} \Downarrow r'}$$

$$\textsc{red-field-1}(\mathsf{f})\quad \frac{}{(H, H_e, \ell_e, \ell), .\mathsf{f} \Downarrow H, H_e, \ell_e, H[\ell][\mathsf{f}]} \quad \ell.\mathsf{f} \in \mathit{dom}^2(H)$$

$$\textsc{red-in}(\mathsf{f}, e)\quad \frac{H, H_e, \ell_e, \ell_c, e \Downarrow r \qquad r, \mathsf{f}\; \mathit{in}_1 \cdot \Downarrow r'}{H, H_e, \ell_e, \ell_c, \mathsf{f}\; \mathit{in}\; e \Downarrow r'}$$

$$\textsc{red-in-1-true}(\mathsf{f})\quad \frac{}{(H, H_e, \ell_e, \ell), \mathsf{f}\; \mathit{in}_1 \cdot \Downarrow H, H_e, \ell_e, 1} \quad \ell.\mathsf{f} \in \mathit{dom}^2(H)$$

$$\textsc{red-in-1-false}(\mathsf{f})\quad \frac{}{(H, H_e, \ell_e, \ell), \mathsf{f}\; \mathit{in}_1 \cdot \Downarrow H, H_e, \ell_e, 0} \quad \ell.\mathsf{f} \notin \mathit{dom}^2(H)$$

2.2 Statements

RED-SKIP

$$\frac{}{H_e, \ell_e, \ell_c, \mathit{skip} \Downarrow H_e, \ell_e, \ell_c}$$

RED-SEQ(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r \qquad r, \cdot ;_1 s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, s_1 ; s_2 \Downarrow r'}$$

RED-SEQ-1(s_2)

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{H_e, \ell_e, \ell_c, \cdot ;_1 s_2 \Downarrow r}$$

RED-ASN(x, e)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathsf{x} :=_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathsf{x} := e \Downarrow r'}$$

RED-ASN-1(x)

$$\frac{\ell_e' = \mathit{fresh}\,(H_e) \qquad E = H_e\,[\ell_e]}{\ell_c, (H_e, \ell_e, v)\,, \mathsf{x} :=_1 \cdot \Downarrow H_e\,[\ell_e' \leftarrow E\,[\mathsf{x} \leftarrow v]]\,, \ell_e', \ell_c} \quad \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_c])$$

RED-ASN-1-LOCAL(x)

$$\frac{\ell_c' = \mathit{fresh}\,(H_e) \qquad C = H_e\,[\ell_c]}{\ell_c, (H_e, \ell_e, v)\,, \mathsf{x} :=_1 \cdot \Downarrow H_e\,[\ell_c' \leftarrow C\,[\mathsf{x} \leftarrow v]]\,, \ell_e, \ell_c'} \quad \mathsf{x} \in \mathit{dom}\,(C)$$

RED-IF(e, s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{if}_1\, s_1\, s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{if}\ (e > 0)\ s_1\, s_2 \Downarrow r'}$$

RED-IF-1-POS(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r}{\ell_c, (H_e, \ell_e, v)\,, \mathit{if}_1\, s_1\, s_2 \Downarrow r} \quad v > 0$$

RED-IF-1-NEG(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{\ell_c, (H_e, \ell_e, v)\,, \mathit{if}_1\, s_1\, s_2 \Downarrow r} \quad v \leqslant 0$$

RED-WHILE(e, s)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{while}_1\ (e > 0)\ s \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{while}\ (e > 0)\ s \Downarrow r'}$$

RED-WHILE-1-NEG(e, s)

$$\frac{}{\ell_c, (H_e, \ell_e, v)\,, \mathit{while}_1\ (e > 0)\ s \Downarrow H_e, \ell_e, \ell_c} \quad v \leqslant 0$$

RED-WHILE-1-POS(e, s)

$$\frac{H_e, \ell_e, \ell_c, s \Downarrow r \qquad r, \mathit{while}_2\ (e > 0)\ s \Downarrow r'}{\ell_c, (H_e, \ell_e, v)\,, \mathit{while}_1\ (e > 0)\ s \Downarrow r'} \quad v > 0$$

RED-WHILE-2(e, s)

$$\frac{H_e, \ell_e, \ell_c, \mathit{while}\ (e > 0)\ s \Downarrow r}{H_e, \ell_e, \ell_c, \mathit{while}_2\ (e > 0)\ s \Downarrow r}$$

RED-RETURN(e)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad r, \mathit{return}_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{return}\, e \Downarrow r'}$$

RED-RETURN-1

$$\frac{}{(H_e, \ell_e, v)\,, \mathit{return}_1 \cdot \Downarrow \mathit{ret}\,(H_e, \ell_e, v)}$$

$$\textsc{red-field-asn}(e_1, \mathtt{f}, e_2)\quad \frac{H, H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, \mathtt{.f} :=_1 e_2 \Downarrow r'}{H, H_e, \ell_e, \ell_c, e_1 \mathtt{.f} := e_2 \Downarrow r'}$$

$$\textsc{red-field-asn-1}(\mathtt{f}, e_2)\quad \frac{H, H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad \ell_c, \ell, r, \mathtt{.f} :=_2 \cdot \Downarrow r'}{\ell_c, (H, H_e, \ell_e, \ell), \mathtt{.f} :=_1 e_2 \Downarrow r'}$$

$$\textsc{red-field-asn-2}(\mathtt{f})\quad \frac{o = H[\ell] \qquad H' = H[\ell \leftarrow o[\mathtt{f} \leftarrow v]]}{\ell_c, \ell, (H, H_e, \ell_e, v), \mathtt{.f} :=_2 \cdot \Downarrow H', H_e, \ell_e, \ell_c} \quad \ell \in dom(H)$$

$$\textsc{red-delete}(e, \mathtt{f})\quad \frac{H, H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, delete_1 \ \mathtt{.f} \Downarrow r'}{H, H_e, \ell_e, \ell_c, delete\ e\mathtt{.f} \Downarrow r'}$$

$$\textsc{red-delete-1}(\mathtt{f})\quad \frac{o = H[\ell] \qquad H' = H[\ell \leftarrow o \setminus \mathtt{f}]}{\ell_c, (H, H_e, \ell_e, \ell), delete_1 \ \mathtt{.f} \Downarrow H', H_e, \ell_e, \ell_c} \quad \ell \in dom(H)$$

2.3 Aborting Rules

$$\textsc{red-error-expr}(e)\quad \frac{}{\sigma, e \Downarrow err} \quad \mathbf{abort}\,\sigma \wedge \neg\mathbf{intercept}_e\,\sigma \qquad\qquad \textsc{red-error-stat}(s)\quad \frac{}{\sigma, s \Downarrow err} \quad \mathbf{abort}\,\sigma$$

$$\frac{\sigma = C[err]}{\mathbf{abort}\,\sigma} \qquad\qquad \frac{}{\mathbf{intercept}_{@_3}\ ret\,(H_e, \ell_e, v)}$$